1 Chernoff/Hoeffding bounds

Let $X_1 \dots X_n \in [0,1]$ be independent random variables, and let $\mu = E[\sum_{i=1}^n X_i]$. Then

- Multiplicative bounds (Chernoff)

  $\Pr\left[\sum X_i - \mu > \lambda\mu\right] \le e^{-\mu\lambda^2/3}$ for $0 < \lambda < 1$ $\quad$ ($\Pr\left[\sum X_i > (1+\lambda)\mu\right] \le e^{-\mu\lambda^2/3}$)

  $\Pr\left[\sum X_i > \lambda\mu\right] \le 2^{-\lambda\mu}$ for $\lambda \ge 6$

  $\Pr\left[\sum X_i - \mu < -\lambda\mu\right] \le e^{-\mu\lambda^2/3}$ for $0 < \lambda < 1$ $\quad$ ($\Pr\left[\sum X_i > (1-\delta)\mu\right] \le e^{-\mu\lambda^2/3}$)

- Additive bounds (Hoeffding)

  $\Pr\left[\left|\sum X_i - \mu\right| \ge \lambda n\right] \le 2e^{-2\lambda^2 n}$, for all $\lambda > 0$.

Modulo constants in the exponent, it's better to use the multiplicative bound. (Additive bound gets tighter exponent, by a constant, for large enough μ.)

2 Approximations/Bounds

- $e^{\frac{x}{(1+x/2)}} \le 1 + x \le e^x$

  $e^{\frac{-x}{(1-x)}} \le 1 - x \le e^{-x}$

- $\ln(1-x) = -(x + x^2/2 + x^2/3 + \dots)$ for any $x > 0$.
- $\ln(1+x) = x - x^2/2 + x^2/3 - x^3/4 + \dots)$ for any $x > 0$.
- Hence $\log_{1+\epsilon} n = O(\frac{1}{\epsilon}\log n)$ for $n \ge 1$ and $\epsilon \in (0,1)$.

3 Gaussians

Say $G : \mathbb{R}^d \to \mathbb{R}^k$ is linear map (matrix) with Gaussians, scaled by $1/\sqrt{k}$. Then, for $\|a\|_2 = 1$, we have for all $D > 0$

$$\Pr[\|Ga - 1\| \ge D] \le \exp[-kD^2/8]$$

and

$$\Pr[\|Ga\| \le 1/D] \le \left(\frac{3}{D}\right)^k$$

(from Naor-Indyk, Nearest Neighbor Preserving Embeddings)